LANDMARK UNIVERSITY, OMU-ARAN

LECTURE NOTE
COLLEGE: COLLEGE OF SCIENCE AND ENGINEERING
DEPARTMENT: MECHANICAL ENGINEERING
Course code: GEC 223
Course title: **FLUID MECHANICS I**
Credit unit: *2 UNITS.*
Course status: *compulsory*
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GEC223 Fluid Mechanics I (2 Units)
Properties of fluids. Fluid statics. Density, pressure, surface tension, viscosity, compressibility, etc. Basic conservation laws, friction effects and losses in laminar and turbulent flows in ducts and pipes. Dimensional analysis and dynamics similitude, principles of construction and operation of selected hydraulic machinery. Hydropower systems. The students should undertake laboratory practical in-line with the topics taught.

Dimensional Analysis.
Dimensional analysis is the process by which the dimensions of equations and physical phenomena are examined to give new insight into their solutions. This analysis can be extremely powerful. Besides being rather elegant, it can greatly simplify problem solving, and for problems where the equations of motion cannot be solved it sets the rules for designing model tests, which can help to reduce the level of experimental effort significantly. The principal aim of dimensional analysis in fluid mechanics is to identify the important non-dimensional parameters that describe any given flow problem. Thus far, we have already encountered a number of non-dimensional parameters, each of which has a particular physical interpretation.
Non-dimensional parameters are widely used in fluid mechanics, and there are good reasons for this.
1. Dimensional analysis leads to a reduced variable set. A problem where the \output" variable, such as the lift force, is governed by a set of (N - 1) \input" variables (for example, a length, a velocity, the density, the viscosity, the speed of sound, a roughness height, etc.), can generally be expressed in terms of a total of (N - 3) non-dimensional groups (for example, the lift coefficient, the Reynolds number, the Mach number, etc.).
2. When testing a scale model of an object, such as a car or an airplane, dimensional analysis provides the guidelines for scaling the results from a model test to the full- scale. In other words, dimensional analysis sets the rules under which full similarity in model tests can be achieved.
3. Non-dimensional parameters are more convenient than dimensional parameters since they are independent of the system of units. In engineering, dimensional equations are sometimes used, and they contribute to confusion, errors and wasted effort. Dimensional equations depend on using the required units for each of the variables, or the answer will be incorrect. They are

common in some areas of engineering, such as in the calculation of heat transfer rates and in describing the performance of turbo-machines.

4. Non-dimensional equations and data presentations are more elegant than their dimensional counterparts. Engineering solutions need to be practical, but they are always more attractive when they display a sense of style or elegance.

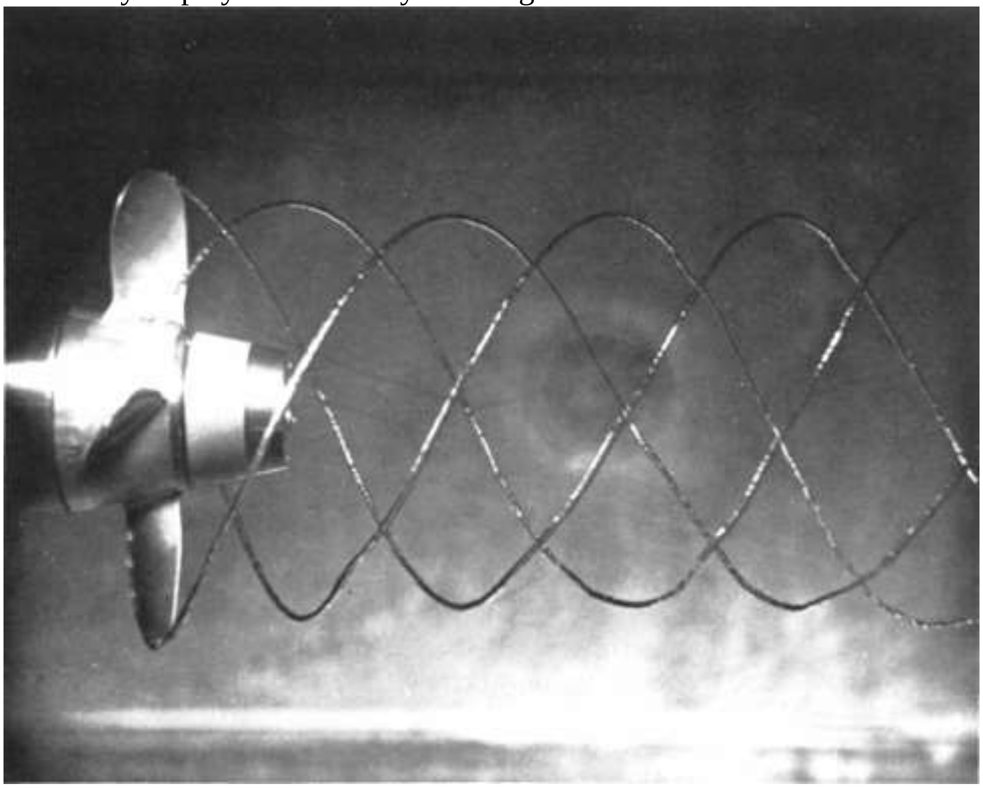

Figure 1: Cavitation on a model propeller. The bubbles are generated near the tip of each blade, and from a helical pattern in the wake. Photograph courtesy of the Garfield Thomas Water Tunnel, Pennsylvania State University.

The most powerful application of dimensional analysis occurs in situations where the governing equations cannot be solved. This is often the case in fluid mechanics. Very few exact solutions of the equations of motion can be found, and for the vast majority of engineering problems involving fluid flows we need to use an approximate analysis where the full equations are simplified to some extent, or we need to perform experiments to determine empirically the behavior of the system empirically over some range of interest (we may, for example, need to understand cavitation on marine propellers, as illustrated in Figure 1). In both cases, dimensional analysis plays a critical role in reducing the amount of effort involved and by providing physically meaningful interpretations for the answers obtained. Instead of solving the equations directly, we try to identify the important variables (such as force, velocity, density, viscosity, the size of the object, etc.), arrange these variables in non-dimensional groups, and write down the functional form of the flow behavior. This procedure establishes the conditions under which similarity occurs, and it always reduces the number of variables that need to be considered. It is rare for dimensional analysis to actually yield the analytical relationship governing the behavior. Usually, it is just the functional form that can be found, and the actual relationship must be determined by experiment. The experiments will also verify if any parameters neglected in the analysis were indeed negligible. To see how dimensional analysis works, we first need to define what system of dimensions we will use, and what is meant by a “complete physical equation."

Dimensional Homogeneity

When we write an algebraic equation in engineering, we are rarely dealing with just numbers. We are usually concerned with quantities such as length, force or acceleration. These quantities have a dimension (e.g., length or distance) and a unit (e.g., inch or meter). In fluid mechanics, the four fundamental dimensions are usually taken to be mass M, length L, time T and temperature θ. Some common variables and their dimensions are as follows (the square brackets are used as shorthand for “the dimensions of .... are").

$$Angular\ velocity = \frac{angular\ measure}{time} = T^{-1}$$

$$Density = \frac{mass}{volume} = ML^{-3}$$

$$Velocity = \frac{length}{time} = LT^{-1}$$

$$Acceleration = \frac{length}{(time)^2} = LT^{-2}$$

$$Force = mass\ x\ acceleration = MLT^{-2}$$

$$Pressure = \frac{force}{area} = ML^{-1}T^{-2}$$

$$Work = force\ x\ distance = ML^2T^{-2}$$

$$Torque = force\ x\ distance = ML^2T^{-2}$$

$$Power = force\ x\ velocity = ML^2T^{-3}$$

$$Dynamic\ viscosity = \frac{stress}{velocity{-}gradient} = \frac{(force/area)}{(velocity/length)} = ML^{-1}T^{-1}$$

$$Kinematic\ viscosity = \frac{viscosity}{density} = \frac{viscosity}{(mass/volume)} = L^2T^{-1}$$

$$Surface\ tension = \frac{force}{length} = MT^{-2}$$

Some quantities are already dimensionless. These include pure numbers, angular degrees or radians, and strain. The concept of a dimension is important because we can only add or compare quantities which have similar dimensions: lengths to lengths, and forces to forces. In other words, all parts of an equation must have the same dimension | this is called the principle of dimensional homogeneity, and if the equation satisfies this principle it is called a complete physical equation. Take, for example, Bernoulli's equation

$$\frac{p}{\rho} + \frac{1}{2}V^2 + gh = B \quad \ldots\ldots \quad 1$$

Where B is a constant. We can examine the dimensions of each term in the equation by writing the dimensional form of the equation:

$$\frac{M}{L^2}\frac{L}{T^2} \times \frac{L^3}{M} + \frac{L^2}{T^2} + \frac{L}{T^2} \times L = [B]$$

(the number $\frac{1}{2}$ is just a counting number with no dimensions). That is,

$$\frac{L^2}{T^2} + \frac{L^2}{T^2} + \frac{L^2}{T^2} = [B]$$

All the parts on the left hand side have the same dimensions of (velocity)^2, and the equation is dimensionally homogeneous. The constant on the right hand side must have the same dimensions as the parts on the left, so that in this case the constant B also has the dimensions of (velocity)^2.
If we rewrote equation 1 as

$$\frac{p}{g\rho} + \frac{1}{2}\frac{V^2}{g} + h = B_1 \text{ or,}$$

$$p + \frac{1}{2}\rho V^2 + \rho g h = B_2$$

then in the first case each term has dimensions of length (including B_1), and in the second case each term has dimensions of pressure (including B_2). Thus we have the principle of dimensional homogeneity

All physically meaningful equations are dimensionally homogeneous.

To put this another way, in order to measure any physical quantity we must first choose a unit of measurement, the size of which depends solely on our own particular preference. This arbitrariness in selecting a unit size leads to the following postulate: any equation that describes a real physical phenomenon can be formulated so that its validity is independent of the size of the units of the primary quantities. Such equations are therefore called complete physical equations. All equations given in this book are complete in this sense. When writing down an equation from memory, it is always a good idea to check the dimensions of all parts of the equation | just to make sure it was remembered correctly. It also helps in verifying an algebraic manipulation or proof where it can be used as a quick check on the answer.

This property of dimensional homogeneity can be useful for:
1. Checking units of equations;
2. Converting between two sets of units;
3. Defining dimensionless relationships (see below).

Results of dimensional analysis

The result of performing dimensional analysis on a physical problem is a single equation. This equation relates all of the physical factors involved to one another. This is probably best seen in an example. If we want to find the force on a propeller blade we must first decide what might influence this force. It would be reasonable to assume that the force, *F*, depends on the following physical properties:
diameter, *d*
forward velocity of the propeller (velocity of the plane), *u*

fluid density, ρ
revolutions per second, N
fluid viscosity, μ
Before we do any analysis we can write this equation:
$F = \phi\ (d, u, \rho, N, \mu)$
or
$0 = \phi_1\ (F, d, u, \rho, N, \mu)$
where ϕ and ϕ_1 are unknown functions.
These can be expanded into an infinite series which can itself be reduced to
$F = K\ d_m\ u_p\ \rho_\theta\ N_r\ \mu_\sigma$
where K is some constant and m, p, q, r, s are unknown constant powers.
From dimensional analysis we
1. obtain these powers
2. form the variables into several dimensionless groups
The value of K or the functions ϕ and ϕ_1 must be determined from experiment. The knowledge of the dimensionless groups often helps in deciding what experimental measurements should be taken.

1.4 Properties of Fluids

The properties outlines below are general properties of fluids which are of interest in engineering. The symbol usually used to represent the property is specified together with some typical values in SI units for common fluids. Values under specific conditions (temperature, pressure etc.) can be readily found in many reference books. The dimensions of each unit is also give in the MLT system.

Dimensional Analysis
In engineering the application of fluid mechanics in designs make much of the use of empirical results from a lot of experiments. This data is often difficult to present in a readable form. Even from graphs it may be difficult to interpret. Dimensional analysis provides a strategy for choosing relevant data and how it should be presented. This is a useful technique in all experimentally based areas of engineering. If it is possible to identify the factors involved in a physical situation, dimensional analysis can form a relationship between them.
The resulting expressions may not at first sight appear rigorous but these qualitative results converted to quantitative forms can be used to obtain any unknown factors from experimental analysis.

Dimensions and units
Any physical situation can be described by certain familiar properties e.g. length, velocity, area, volume, acceleration etc. These are all known as dimensions. Course dimensions are of no use without a magnitude being attached. We must know more than that something has a length. It must also have a standardised unit - such as a meter, a foot, a yard etc. Dimensions are properties which can be measured. Units are the standard elements we use to quantify these dimensions. In dimensional analysis we are only concerned with the nature of the dimension i.e. its quality not its quantity. The following common abbreviations are used:

length = L
mass = M
time = T
force = F
temperature = Q
In this module we are only concerned with L, M, T and F (not Q). We can represent all the physical properties we are interested in with L, T and one of M or F (F can be represented by a combination of (LTM). These notes will always use the LTM combination.

The following table lists dimensions of some common physical quantities:

Quantity	SI Unit		Dimension
velocity	m/s	ms^{-1}	LT^{-1}.
acceleration	m/s^2	ms^{-2}	LT^{-2}.
force	N kgm/s^2	$Kgms^{-2}$	MLT^{-2}.
energy (or work)	Joule J Nm, Kgm^2/s^2	$Kgm2s^{-2}$	ML^2T^{-2}.
power	Watt W Nm/s Kgm^2/s^3	Nms^{-1} $Kgm2s^{-3}$	ML^2T^{-3}.
pressure (or stress)	Pascal P, N/m^2, $Kg/m/s^2$	Nm^{-2} $Kgm^{-1}s^{-2}$	$ML^{-1}T^{-2}$
density	Kg/m^3	Kgm^{-3}	ML^{-3}
specific weight	N/m^3 $Kg/m^2/s^2$	$Kgm^{-2}s^{-2}$	$ML^{-2}T^{-2}$
relative density	a ratio no units		1 no dimension
viscosity	$N\ s/m^2$ Kg/m s	$N\ s\ m^{-2}$ $Kg\ m^{-1}\ s^{-1}$	$ML^{-1}T^{-1}$
surface tension	N/m Kg/s^2	Nm^{-1} $Kg\ s^{-2}$	MT^{-2}

1.4.1 Density

The density of a substance is the quantity of matter contained in a unit volume of the substance. It can be expressed in three different ways.

1.4.1.1 Mass Density (Density or Specific density)

Mass Density, ρ , (rho) is defined as the mass of substance per unit volume (m/V); at a standard temperature and pressure
Units: Kilograms per cubic metre, kg / m^3(or kgm^{-3})
Dimensions: ML^{-3}
Typical values:

Water = 1000 kgm^{-3}, Mercury = 13546 kgm^{-3}Air = 1.23 kgm^{-3}, Paraffin Oil = 800 kgm^{-3}.
(at pressure =1.013×10^{-5}N $m^{-2}$and Temperature = 288.15 K.)

1.4.1.2 Specific Weight

Specific Weight ω , (sometimes γ, and sometimes known as specific gravity) is defined as the weight per unit volume at a standard temperature and pressure. or
The force exerted by gravity, g, upon a unit volume of the substance. The Relationship between g and ω can be determined by Newton's 2^{nd}Law, since weight per unit volume = mass per unit volume × g
$\omega = \rho g$
Units: Newton's per cubic metre, N / m^3(or N m^{-3})
Dimensions: $ML^{-2}T^{-2}$.
Typical values:
Water =9814 N m^{-3}, Mercury = 132943 N m^{-3}, Air =12.07 N m^{-3}, Paraffin Oil =7851 N m^{-3}

1.4.1.3 Relative Density

Relative Density, σ , is defined as the ratio of mass density of a substance to some standard mass density. For solids and liquids this standard mass density is the maximum mass density for water (which occurs at 4c) at atmospheric pressure.

$$\sigma = \frac{\sigma_{substance}}{\sigma_{H_2O(at 4^0C)}}$$

Or
The specific gravity S of a liquid is the ratio of its density to that of water at 4^0C and the specific gravity of a gas is the ratio of its density to that of air at STP; water and air being accepted as the reference liquid and gaseous fluid respectively.

$$S = \frac{specific\ weight\ of\ liquid}{specific\ of\ water} = \frac{w_{liquid}}{w_{water}}$$

Units: None, since a ratio is a pure number.
Dimensions: 1.
Typical values: Water = 1, Mercury = 13.5, Paraffin Oil =0.8.

1.4.1.4 Specific Volume

It is customary in the study of thermodynamics to use the terms 'specific volume'. The specific volume is the volume per unit mass of the fluid. It follows, therefore, that the specific volume v is the inverse of the mass density ρ; $v = \frac{1}{\rho}$

1.4.2 Viscosity

Viscosity, μ, is the property of a fluid, due to cohesion and interaction between molecules, which offers resistance to sheer deformation. Different fluids deform at different rates under the same shear stress.
Fluid with a high viscosity such as syrup, deforms more slowly than fluid with a low viscosity such as water.

All fluids are viscous, “Newtonian Fluids” obey the linear relationshipgiven by Newton’s law of viscosity.

$\tau = \mu \frac{du}{dy}$, which we saw earlier.

Where τ is the shear stress,
Units N m^{-2}; kg $m^{-1}s^{-2}$
Dimensions $ML^{-1}T^{-2}$.$\frac{du}{dy}$ is the velocity gradient or rate of shear strain, and has
Units: radians s^{-1},
Dimensions t^{-1}
μ is the “coefficient of dynamic viscosity” - see below.

1.4.2.1 Coefficient of Dynamic Viscosity

The Coefficient of Dynamic Viscosity, μ , is defined as the shear force, per unit area, (or shear stress τ), required to drag one layer of fluid with unit velocity past another layer a unit distance away.

$$\mu = \tau \Big/ \frac{du}{dy} = \frac{Force}{Area} \Big/ \frac{Velocity}{Distance} = \frac{Force \times Time}{Area} = \frac{Mass}{Length \times Area}$$

Units: Newton seconds per square metre, N sm^{-2}or Kilograms per meter per second, $kgm^{-1}s^{-1}$.(Although note that μ is often expressed in Poise, P, where 10 P = 1 $kgm^{-1}s^{-1}$.)
Typical values:
Water =$1.14 \times 10^{-3} kgm^{-1}s^{-1}$, Air =$1.78 \times 10^{-5} kgm^{-1}s^{-1}$, Mercury =1.552 $kgm^{-1}s^{-1}$,
Paraffin Oil =1.9 $kgm^{-1}s^{-1}$.

1.4.2.2 Kinematic Viscosity

Kinematic Viscosity, ν , is defined as the ratio of dynamic viscosity to mass density.

$$\nu = \frac{\mu}{\rho}$$

Units: square metres per second, m^2s^{-1}
(Although note that ν is often expressed in Stokes, St, where 10^4St = 1 m^2s^{-1}.)
Dimensions: L^2T^{-1}.
Typical values:
Water =$1.14 \times 10^{-6} m^2s^{-1}$, Air =$1.46 \times 10^{-5} m^2s^{-1}$, Mercury =$1.145 \times 10^{-4} m^2s^{-1}$,
Paraffin Oil =$2.375 \times 10^{-3} m^2s^{-1}$.